Spherically Symmetric Solutions of Gravitational Field Equations in Kalb-Ramond Background

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Static spherically symmetric solution in a background spacetime with torsion is derived explicitly. The torsion considered here is identified with the field strength of a second rank antisymmetric tensor field namely the Kalb-Ramond field and the proposed solution therefore has much significance in a string inspired gravitational field theory.

I. INTRODUCTION

The extension of the geometric principles of general relativity to the physics at a microscopic level where matter formation is done by elementary particles, characterized by a spin angular momentum in addition to the mass, is achieved in Einstein-Cartan theory. In such a theory the symmetric Christoffel connection is modified with the introduction of an antisymmetric tensorial term, known as the spacetime torsion, which is presumed to have a direct relationship with spin [1–3]. Over the years, since Cartan's pioneering works in early 1920s, there had been numerous interesting questions especially as to how the introduction of torsion in spacetime affects the solutions of the gravitational field equations under various circumstances. The advent of string theory and its emergence as a powerful tool for the quantum theoretic unification of all the fundamental forces of nature [4] has brought a resurgence of interest in spacetimes with torsion and the need for getting satisfactory answers to the aforesaid questions is enhanced. The field strength corresponding to the second rank antisymmetric tensor in the string spectrum is identified as the space time torsion.

In this paper, we aim to study the possible static (i.e., time-invariant) spherically symmetric solutions of the gravitational field equations in presence of torsion in spacetime. One of the important motivations in looking for such solutions is ideally to realise the effect of torsion on the electromagnetic waves coming from distant galactic sources [5]. The basic underlying theory adopted by us is that proposed by Majumdar and SenGupta [6], in which a new antisymmetric tensor field $B_{\mu\nu}$, identified as the Kalb-Ramond (KR) field, is introduced to ensure the U(1) gauge-invariance of the electromagnetic theory in a background with torsion. The massless KR field is taken to be the possible source of torsion. The strength of this KR field $H_{\mu\nu\lambda} = \partial_{[\mu}B_{\nu\lambda]}$ is modified by U(1) Chern-Simons terms originating from the quantum consistency of an underlying string theory and the coupling of the KR field with torsion is done in a way that the gauge symmetry is preserved in the resulting action.

In Section 2, we focus our attention to the action of a purely gravitational field theory in a torsioned background. We vary the action, with respect to the metric $g_{\mu\nu}$ and as well as to the Kalb-Ramond field $B_{\mu\nu}$, to obtain two sets of field equations. The variation of the action with respect to $g_{\mu\nu}$ merely gives a modified version of the old Einstein's equations. The modification comes in a way that the coupling of torsion with gravity is reflected in the spin-density tensor, just as matter coupling to gravity is reflected in the energy-momentum tensor in the old equations. In Section 3, we seek the possible spherically symmetric solutions of the field equations in vacuum and show that it is not necessarily static, unlike what we get in Schwarzschild solution in absence of torsion. However in section 4, we construct an explicit static spherically symmetric solution with an appropriate Kalb-Ramond background.

II. FIELD EQUATIONS FOR GAUGE-INVARIANT EINSTEIN-CARTAN-KALB-RAMOND COUPLING

The action is taken to be of the form [6]:

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$$S = \int d^4x \sqrt{-g} \left[\frac{\tilde{R}(g,T)}{\kappa} - \frac{1}{2} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + \frac{1}{\sqrt{\kappa}} T^{\mu\nu\lambda} H_{\mu\nu\lambda} \right] \tag{1}$$

where $\tilde{R}(g,T)$ is the scalar curvature for the Einstein-Cartan spacetime where the connections contain torsion $T^\alpha_{\mu\nu}$:

$$\tilde{\Gamma}^\alpha_{\mu\nu} = \Gamma^\alpha_{\mu\nu} - T^\alpha_{\mu\nu} , \tag{2}$$

and $\kappa = 16\pi G$ is the coupling constant. We choose $T^{\alpha\mu\nu}$ to be antisymmetric in its all three indices.

Direct calculation shows that the curvature scalar $\tilde{R}(g,T)$ of the Einstein-Cartan spacetime is related to that of purely Riemannian spacetime by

$$\tilde{R}(g,T) = R(g) + T_{\mu\nu\lambda} T^{\mu\nu\lambda}. \tag{3}$$

Moreover, the torsion tensor $T_{\mu\nu\lambda}$, being an auxiliary field in Eq.(1), obeys the constraint equation

$$T_{\mu\nu\lambda} = -\frac{\sqrt{\kappa}}{2} H_{\mu\nu\lambda} \tag{4}$$

which implies that the augmented KR field strength 3-tensor plays the role of the spin angular momentum density, considered to be the source of torsion [1].

Substituting Eq.(4) in the action (1) and varying the latter with respect to $g_{\mu\nu}$ and $B_{\mu\nu}$ respectively, we obtain the field equations

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \tau_{\mu\nu} \tag{5}$$

and

$$D_\mu H^{\mu\nu\lambda} \equiv \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} H^{\mu\nu\lambda}) = 0 \tag{6}$$

where $R_{\mu\nu}$ and $G_{\mu\nu}$ are respectively the Ricci tensor and the Einstein tensor of Riemannian geometry; and $\tau_{\mu\nu}$ is a symmetric 2-tensor, having direct analogy with the energy-momentum tensor, and is a clear manifestation of the spin-density tensor in the field equations for the Einstein-Cartan spacetime. The expression for $\tau_{\mu\nu}$ is of the form:

$$\tau_{\mu\nu} = \frac{1}{\sqrt{-g}} \left[\frac{\partial(\sqrt{-g}\mathcal{L}_{KR})}{\partial g^{\mu\nu}} - \partial_\alpha \left(\frac{\partial(\sqrt{-g}\mathcal{L}_{KR})}{\partial(\partial_\alpha g^{\mu\nu})} \right) \right] \tag{7}$$

where $\mathcal{L}_{KR}$ is the Lagrangian density due to the KR field, which can be expressed as

$$\mathcal{L}_{KR} = \frac{3}{4} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma}. \tag{8}$$

Substituting Eq.(8) in Eq.(7), we finally obtain

$$\tau_{\mu\nu} = \frac{3}{4} \left(3 g_{\nu\rho} H_{\alpha\beta\mu} H^{\alpha\beta\rho} - \frac{1}{2} g_{\mu\nu} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right). \tag{9}$$

Now, by virtue of the symmetric nature of $G_{\mu\nu}$ and $\tau_{\mu\nu}$, Eq.(5) can have, in general, ten component equations. Also, the totally antisymmetric property of $H_{\mu\nu\lambda}$ implies that it has four independent components, viz., $H_{012}, H_{013}, H_{023}$ and H_{123}, which we denote, for simplicity, as h_1, h_2, h_3 and h_4 respectively; the corresponding contravariant components are denoted by h^1, h^2, h^3 and h^4 respectively. Eq.(6) is a set of six independent equations

$$D_2 h^1 + D_3 h^2 = 0 \tag{10}$$
$$D_1 h^1 - D_3 h^3 = 0 \tag{11}$$
$$D_1 h^2 + D_2 h^3 = 0 \tag{12}$$
$$D_0 h^1 + D_3 h^4 = 0 \tag{13}$$
$$D_0 h^3 + D_1 h^4 = 0 \tag{14}$$
$$D_0 h^2 - D_2 h^4 = 0. \tag{15}$$

III. STATIC SPHERICALLY SYMMETRIC SOLUTIONS

The line element is taken in its most general spherical symmetric form:

$$ds^2 = e^{\nu(r,t)}dt^2 - e^{\lambda(r,t)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{16}$$

so that the metric tensor is

$$g_{\mu\nu} = diag(e^\nu, -e^\lambda, -r^2, -r^2\sin^2\theta). \tag{17}$$

The first set of field equations (5) take the form:

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = \bar{\kappa}(h_1h^1 + h_2h^2 + h_3h^3 - h_4h^4) \tag{18}$$

$$e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = \bar{\kappa}(h_1h^1 + h_2h^2 - h_3h^3 + h_4h^4) \tag{19}$$

$$e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu'-\lambda'}{r}\right) - e^{-\nu}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) = 2\bar{\kappa}(h_1h^1 - h_2h^2 + h_3h^3 + h_4h^4) \tag{20}$$

$$e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu'-\lambda'}{r}\right) - e^{-\nu}\left(\ddot{\lambda} + \frac{\dot{\lambda}^2}{2} - \frac{\dot{\lambda}\dot{\nu}}{2}\right) = 2\bar{\kappa}(-h_1h^1 + h_2h^2 + h_3h^3 + h_4h^4) \tag{21}$$

$$\frac{e^{-\lambda}\dot{\lambda}}{r} = 2\bar{\kappa}h_3h^4 \tag{22}$$

$$h_2h^4 = h_1h^4 = h_2h^3 = h_1h^2 = h_3h^1 = 0 \tag{23}$$

and the other set (10) - (15) reduce to:

$$h^1{}_{,2} + \cot\theta\, h^1 = -h^2{}_{,3} \tag{24}$$

$$h^1{}_{,1} + \left(\frac{\nu'+\lambda'}{2} + \frac{2}{r}\right)h^1 = h^3{}_{,3} \tag{25}$$

$$h^2{}_{,1} + \left(\frac{\nu'+\lambda'}{2} + \frac{2}{r}\right)h^2 = -h^3{}_{,2} - \cot\theta\, h^3 \tag{26}$$

$$h^1{}_{,0} + \left(\frac{\dot{\nu}+\dot{\lambda}}{2}\right)h^1 = -h^4{}_{,3} \tag{27}$$

$$h^3{}_{,0} + \left(\frac{\dot{\nu}+\dot{\lambda}}{2}\right)h^3 = -h^4{}_{,1} - \left(\frac{\nu'+\lambda'}{2} + \frac{2}{r}\right)h^4 \tag{28}$$

$$h^2{}_{,0} + \left(\frac{\dot{\nu}+\dot{\lambda}}{2}\right)h^2 = h^4{}_{,2} + \cot\theta\, h^4. \tag{29}$$

Here dot and prime respectively stand for the differentiations with respect to t and r; and $\bar{\kappa} = \frac{9\kappa}{4}$.

The left hand sides of Eqs.(20) and (21) are identical, which implies that both h_1 and h_2 must vanish, and this will satisfy Eq.(23) as well. It then follows from Eqs.(25) and (27) that

$$h^3{}_{,3} = h^4{}_{,3} = 0 \tag{30}$$

which means that h^3 and h^4 are both independent of the coordinate ϕ. But from Eq.(22) we find that $\dot{\lambda}$ is, in general, non-zero, i.e., λ is time-dependent. Thus we arrive at an important result: the spherically symmetric gravitational field in vacuum, in a spacetime with torsion, is not necessarily static, unlike the case in the Schwarzschild solution of Einstein's field equations. As we are primarily interested in static vacuum solutions in torsioned background, we set $\dot{\lambda} = \dot{\nu} = 0$, which means that both λ and ν are time-invariant and Eq.(22) suggests that either h_3 or h_4 or both should be equal to zero. But if both h_3 and h_4 vanish, i.e., all the torsion components are zero, we get back the Schwarzschild metric; and since we are interested in solutions involving torsion, we are bound to make the following two choices:

A. Choice I : $h_3 \neq 0, h_4 = 0$

In this case the field equations take the form:

$$e^{-\lambda} \left(\frac{1}{r^2} - \frac{\lambda'}{r} \right) - \frac{1}{r^2} = \bar{\kappa} h_3 h^3 \tag{31}$$

$$e^{-\lambda} \left(\frac{1}{r^2} + \frac{\nu'}{r} \right) - \frac{1}{r^2} = -\bar{\kappa} h_3 h^3 \tag{32}$$

$$e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu' - \lambda'}{r} \right) = 2\bar{\kappa} h_3 h^3 \tag{33}$$

$$h^3{}_{,2} + \cot\theta \ h^3 = 0 \tag{34}$$

$$h^3{}_{,0} = h^3{}_{,3} = 0. \tag{35}$$

The left hand sides of Eqs.(31) - (33) are all functions of r only, therefore, their right hand sides consisting of $h_3 h^3$ must also be functions of r only. It follows from Eq.(35) that h_3 (and hence h^3) is not only independent of ϕ, but also independent of t.

Now,

$$h_3 \equiv H_{023} = g_{00} \ g_{22} \ g_{33} \ H^{023}$$
$$= e^{\nu} r^4 \sin^2\theta \ h^3. \tag{36}$$

Hence,

$$h_3 h^3 = e^{\nu} r^4 \sin^2\theta \ (h^3)^2. \tag{37}$$

On integration, Eq.(34) yields

$$h^3(r, \theta) = \frac{h^3(r)}{\sin\theta}. \tag{38}$$

Substituting Eq.(38) in Eq.(37), we get

$$h_3 h^3 = e^{\nu} r^4 [h^3(r)]^2 \tag{39}$$

which indicates that $h_3 h^3$ is indeed a function of r only.

We denote

$$h_3 h^3 \equiv [h(r)]^2 \tag{40}$$

and write the field equations in a more convenient way so that they can be solved easily:

$$\frac{d}{dr} \left(re^{-\lambda} \right) = 1 + \bar{\kappa} r^2 h^2 \tag{41}$$

$$(re^{-\lambda})\{\nu' + \frac{d}{dr}[\ln(r^2 e^{-\lambda})]\} = 2 \tag{42}$$

$$e^{-\lambda} \left(\nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu' - \lambda'}{r} \right) = 2\bar{\kappa} r^2 h^2 \tag{43}$$

where the second equation, i.e., Eq.(42), is obtained simply by adding Eqs.(31) and (32). Eqs.(41) and (42) can be solved simultaneously to obtain

$$e^{-\lambda} = 1 + \frac{c_1}{r} + \frac{\tau(r)}{r} \tag{44}$$

and

$$e^{\nu} = \frac{c_2}{r(r + \tau(r) + c_1)} \exp\left[\int \frac{2dr}{r + \tau(r) + c_1} \right] \tag{45}$$

where c_1 and c_2 are the constants of the integrations and

$$\tau(r) = \bar{\kappa} \int r^2 h^2(r) dr. \tag{46}$$

For vanishing torsion (i.e., $\tau(r) = 0$), these solutions reduce exactly to the Schwarzschild solution, viz., $e^\nu = e^{-\lambda} = 1 - \frac{r_s}{r}$, provided $c_2 = 1$ and $c_1 = -r_s$, where $r_s = 2GM$ is the Schwarzschild radius. However, when torsion is non-zero, the acceptability of these solutions rests on two factors: firstly, despite satisfying Eqs.(41) and (42), they must satisfy Eq.(43) as well, which is a separate field equation involving ν and λ; and secondly, they must attain the asymptotic forms $e^\nu = e^\lambda = 1$ in the limit $r \to \infty$. Both these conditions may be fulfilled only with some suitable form of $\tau(r)$. We explore such a possibility in Section 4.

B. Choice II : $h_4 \neq 0, h_3 = 0$

The field equations for this choice are

$$e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2} = -\bar{\kappa} h_4 h^4 \tag{47}$$

$$e^{-\lambda}\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = \bar{\kappa} h_4 h^4 \tag{48}$$

$$e^{-\lambda}\left(\nu'' + \frac{\nu'^2}{2} - \frac{\nu'\lambda'}{2} + \frac{\nu' - \lambda'}{r}\right) = 2\bar{\kappa} h_4 h^4 \tag{49}$$

$$h^4{}_{,1} + \left(\frac{\nu' + \lambda'}{2} + \frac{2}{r}\right) h^4 = 0 \tag{50}$$

$$h^4{}_{,2} + \cot\theta \; h^4 = 0 \tag{51}$$

$$h^4{}_{,3} = 0. \tag{52}$$

Here, again we find from Eqs.(47) - (49) that $h_4 h^4$ must have to be a function of r only, i.e., independent of t, θ and ϕ. Eq.(52) shows that h_4, and hence h^4, is independent of ϕ, but there is no equation from which it is evident that $h_4 h^4$ is independent of t. Meanwhile, we have

$$h_4 \equiv H_{123} = g_{11} \, g_{22} \, g_{33} \, H^{123}$$
$$= -e^\lambda r^4 \sin^2\theta \; h^4 \tag{53}$$

which means

$$h_4 h^4 = -e^\lambda r^4 \sin^2\theta \; (h^4)^2. \tag{54}$$

Integrating Eq.(51), we get

$$h^4(r, \theta, t) = \frac{h^4(r, t)}{\sin\theta}. \tag{55}$$

This on substitution into Eq.(54), yields

$$h_4 h^4 = -e^\lambda r^4 [h^4(r, t)]^2 \tag{56}$$

which is independent of θ. We assume that $h_4 h^4$ is independent of t as well, so that a static spherically symmetric solution can be obtained:

$$h_4 h^4 \equiv [\bar{h}(r)]^2. \tag{57}$$

The r-dependence of h^4 could be found directly from Eq.(50):

$$h^4(r, t, \theta) = \frac{h^4(t, \theta)}{r^2} e^{-\frac{\nu + \lambda}{2}} \tag{58}$$

so that

$$h_4 h^4 = -e^{-\nu}[h^4(r,t)]^2 = -\alpha e^{-\nu} \tag{59}$$

where we take $[h^4(t)]^2 = \alpha$ (a constant), i.e., h^4 is kept time-invariant so as that $h_4 h^4$ remains independent of time. Thus

$$\bar{h}(r) = i \sqrt{\alpha} \, e^{-\nu/2} \tag{60}$$

But this is quite unrealistic and not acceptable since $e^{-\nu}$, identified as g^{00}, must approach unity in the asymptotic limit $r \to \infty$, leading to a finite non-vanishing value of torsion in that limit as is evident from Eq.(59). Moreover, Eq.(47) gives the result

$$g^{00} \equiv e^{-\nu} = \frac{\frac{d}{dr}(re^{-\lambda}) - 1}{\bar{\kappa}\alpha r^2}, \tag{61}$$

wherefrom it is clear that in order to make g^{00} approach unity asymptotically as $r \to \infty$ we must choose

$$\frac{d}{dr}(re^{-\lambda}) = 1 + \bar{\kappa}\alpha r^2 \gamma(r) + \delta(r), \tag{62}$$

and impose the conditions on the arbitrary functions $\gamma(r)$ and $\delta(r)$ that they must approach 1 and 0 respectively in the $r \to \infty$ limit. But this is simply not possible since this implies that $e^{-\lambda}$, i.e., $-g^{11}$ tends to infinity (instead of approaching unity) as $r \to \infty$.

Hence, we disregard the choice II and infer that the choice I leads to the only possibility of getting a generalized spherically symmetric solution of the gravitational field equations in a curved spacetime with a Kalb-Ramond background.

IV. EXACT STATIC SPHERICALLY SYMMETRIC SOLUTIONS FOR THE METRIC AND THE KALB-RAMOND BACKGROUND

A close insight to the field equations (41) - (43) reveals that the solutions (44) and (45), which were obtained by solving Eqs. (41) and (42) only, would satisfy Eq.(43) as well, only when the function $\tau(r)$, which depends on the KR field strength $h(r)$, satisfies the equation

$$\tau'' + \frac{\tau'}{r} = \frac{\tau'(\tau' - 1)}{r + c_1 + \tau} \tag{63}$$

which, on first integration, yields

$$\tau' = \frac{\alpha}{r}(r + c_1 + \tau) \exp\left[-\int \frac{2 \, dr}{r + c_1 + \tau}\right] \tag{64}$$

where α is an integration constant.

The above equation has an exact solution

$$\tau(r) = \frac{\alpha}{r^2} \tag{65}$$

for $c_1 = 0$, whence we obtain the solution consistent with all the three field equations (41) - (43) :

$$e^{-\lambda} = 1 - \frac{\alpha}{r^2} \tag{66}$$

$$e^{\nu} = 1. \tag{67}$$

Here we note that the constant c_2 in eq.(45) is taken to be equal to 1 in order that e^{ν} remains unity when $r \to \infty$. Moreover, the form of $\tau(r)$ in Eq.(65) suggests that the KR field strength has an inverse square dependence on r:

$$h(r) = \sqrt{\frac{\alpha}{\bar{\kappa}}} \frac{1}{r^2}. \tag{68}$$

But depending on the sign of α, we have the following two cases:

Case I : For real and positive α the KR field h is real and we cannot find any event horizon for the metric under consideration. This implies that the singularity at $r = 0$ is, actually, a naked singularity, not being shielded by an event horizon as in the ideal Schwarzschild case. In fact, there exists another singularity at $r = \sqrt{\alpha}$, whose origin is not quite clear, at least physically, but it is certain that its presence cannot lead to an event horizon.

Case II : For real and negative α the KR field h is imaginary, but again we find the absence of an event horizon leads to a naked singularity at $r = 0$. Moreover, we note that, unlike the case I, we do not have any other singularity in this case.

V. CONCLUSIONS

We, therefore, have shown that in a spherically symmetric Kalb-Ramond background one may obtain a static spherically symmetric solution for the gravitational field equations. The KR field, modified suitably by Chern-Simons terms for gauge-invariance, may couple with electromagnetic field to produce non-trivial effects like optical activity for the radiowaves from the distant galaxies [7,8]. In the references cited, the calculations have been done for a flat universe and more generally for matter and radiation dominated universe. However, our work will now allow to estimate such effects in static spherically symmetric background with KR field.

As the augmented KR background can be obtained in the scenario of string theory in the form of a second-rank antisymmetric tensor field [4] from a completely different angle like anomaly cancellation, our work therefore proposes a static spherically symmetric solution for string inspired gravity. In string theory the field strength corresponding to the second rank antisymmetric tensor field (which is identified with spacetime torsion) is related to the axion through a duality. Our work therefore essentially exhibits the existence of static spherically symmetric solution in an axion background. This solution now opens new possibilities of exploring various aspects of the gravitational solution including black holes. Work in this direction is in progress. Furthermore, the nature of the nonstatic solutions and their implications must also be investigated to have a complete understanding of the spherically symmetric solution in a torsioned background. We also propose to extend this work in a more generalized scenario where the Kalb-Ramond background admits of a parity violating extension [9]. We expect to have some interesting solutions in such a case which may offer an explanation to the parity violating phenomenon in such a spacetime.

It must also be pointed out here that the horizon-free solution obtained for the inverse square nature of the KR field is completely compatible with the "no hair" conjecture [10]. That is to say this form of KR field exhibits an important feature in having no "hair" whatsoever.

VI. ACKNOWLEDGEMENT

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